

Modelling the Covid-19 Outbreak using SIR Model

Aryaman Sharma

Student, The Doon School, Dehradun, India
E-mail: aryaman.43.2021@doonschool.com

Abstract—In this paper, the dynamics of the COVID-19 pandemic are explored by examining and studying the SIR model primarily by finding analytical solutions to the epidemic model. Firstly, I modelled the SIR curves by the traditional method by devising differential equations and consequently finding the data. I used this method to model curves for t -step-value of 1. Moving forward, I used the Initial Value Problem or Euler's Method to model the curves for t -step-value of 5 and 10. The resulting models for the novel coronavirus outbreak are then compared and evaluated on the bases of various parameters.

Keywords: SIR Model; Initial Value Problem; Euler's Method; COVID-19.

INTRODUCTION

On December 31, 2019, the first case of COVID-19 was reported in Wuhan City, Hubei province, China. The Wuhan Municipal Health Commission reported a sum of 27 pneumonia cases of unknown causes with a common link to the Wuhan's Huanan Seafood Wholesale Market. According to the health commission, samples from the seafood market tested positive for novel coronavirus. Hence, the market was closed down on January 1, 2020. By January 20, 2020, the virus had spread to three countries outside of China: Thailand, Japan and South Korea. Wuhan City's lockdown on January 23, 2020 was followed by an announcement by the World Health Organisation (WHO), which declared the outbreak of COVID-19 also known as SARS-CoV-2 as a "public health emergency of international concern" [1]. This was when things started turning pretty bad. International airlines started cancelling flights and also commenced testing at airports. There are over 2,387,502 confirmed, 164,194 deaths and 613,443 recovered cases of novel coronavirus worldwide as of April 19, 2020 [2].

COVID-19 is caused by a completely new type of 'coronavirus' which was named by the World Health Organization (WHO) as 2019-nCoV. It is the seventh member of the coronavirus family, together with MERS-nCoV and SARS-nCoV, that can spread to humans. The symptoms of the infection include high fever, painful cough, shortness of breath, and sometimes even diarrhoea. In more severe cases, the symptoms extend to the causation of pneumonia and even death [3]. The incubation period, i.e. the period between exposure to an infection and the appearance of the first

symptoms, can last for 2 weeks or sometimes even longer. Due to the highly contagious nature of the virus, it can spread to other people mainly through droplet transmission and close contact [4].

As of April 16, 2020, the novel coronavirus has spread to more than 180 countries including China [5]. In countries like The United States of America and Spain, the affected cases are touching the ceiling by measuring 764,265 and 198,674 respectively [2]. At this point of time, for the better predictability and understanding of this outbreak, I decided upon writing a research paper on modelling the SIR epidemiological model for China.

AIM

My aim from this exploration is modelling the epidemic of the COVID-19 using various mathematical modelling techniques like the Differential Equations, Euler's Method and Differential Transformation Method. Through this exploration I wish to make the general affected population more aware of the situation we all are trapped in.

PRELIMINARY TRIALS

Before taking the approach of the SIR model, I used the exponent model to model and observe my data. I used the following data from Wuhan, China, of infected and recovered for the first four days to derive an exponential equation [6].

Table 1: Infected and Recovered+Deaths in China for the first 4 days.

Days	Infected	Recovered+Deaths
1	41	2
2	45	2
3	62	2
4	121	3

Let $f(x)$ be the exponential function modelling the infected population of COVID-19.

Hence, $f(x) = ae^{bt} + c$

Where a , b and c are constants.

We can derive the following equations using Table 1:

Day 1:

$$41 = ae^{b \times 1} + c \tag{1}$$

Day 2:

$$45 = ae^{b \times 2} + c \tag{2}$$

Day 3:

$$62 = ae^{b \times 3} + c \tag{3}$$

Day 4:

$$121 = ae^{b \times 4} + c \tag{4}$$

Now, solving eq1 and eq2 simultaneously:

$$45 = ae^{b \times 2} + c$$

$$41 = ae^{b \times 1} + c$$

$$4 = ae^{2b} - ae^b$$

$$\text{Therefore, } 4 = ae^b(e^b - 1) \tag{5}$$

Now, solving eq3 and eq4 simultaneously:

$$121 = ae^{b \times 4} + c$$

$$62 = ae^{b \times 3} + c$$

$$59 = ae^{4b} - ae^{3b} \tag{6}$$

$$\text{Therefore, } 59 = ae^{3b}(e^b - 1) \tag{6}$$

Now, divide eq6 by eq5 to solve for b:

$$\frac{59}{4} = \frac{ae^{3b}(e^b - 1)}{ae^b(e^b - 1)}$$

$$\frac{59}{4} = e^{2b}$$

$$\ln \frac{59}{4} = \ln e^{2b}$$

$$\ln \frac{59}{4} = 2b$$

$$b = \frac{1}{2} \ln \frac{59}{4}$$

$$b = 1.35$$

Now, putting in the value of b in eq6 and evaluating for a:

$$59 = ae^{3 \times 1.35}(e^{1.35} - 1)$$

$$59 = ae^{4.05}(e^{1.35} - 1)$$

$$a = 0.34$$

Now, putting in the values of a and b in eq 1 and evaluating for c:

$$41 = 0.34 \times e^{1.35} + c$$

$$41 - 0.34 \times e^{1.35} = c$$

$$c = 39.7$$

Therefore, the function $f(t) = 0.34e^{1.35t} + 39.7$ is the model for infected population with time t . The function is graphed in Figure 1.

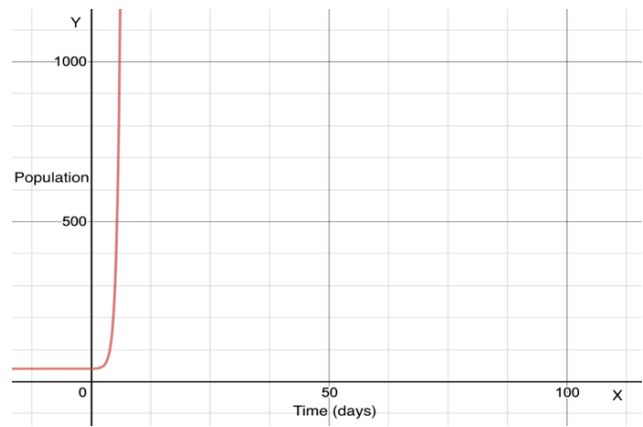


Figure 1: Exponential Function Graph for Population-Time(days)

As it can be seen in Figure 1, the infected population keeps increasing exponentially with an increment in the time in days. This is only virtually possible, because at some point of time, the affected country's governments or the population themselves will take precautionary measures, like the government could place a lockdown as a result ceasing most economic and day-to-day work or activities in order to prevent more transmission. Including this, the general population would also start wearing gloves and masks to protect themselves from the pandemic. These measures are not taken into account in the exponent model. Hence, after a round of preliminary trials, I planned on using the SIR curve and various differential equations to model the COVID 19 outbreak.

THE SIR MODEL

4.1 Background

The SIR model, also known as the Kermack-McKendrick model [7] is an epidemiological model that determines by assessing the theoretical number of people infected with a certain disease or illness over time [8]. It is essential to notice that the disease or illness should be contagious. SIR refers to Susceptible, Infected and Recovered or Removed respectively. Susceptible refers to set of the population which can catch the infection and hence may become infected if exposed to the virus, Infectious is the set of population which shows signs of infection and can transmit the virus if anyone from the susceptible set is exposed to them, and Removed or Recovered is the set of population which was previously infected but is now no longer infectious and are also, most importantly immune to the disease or illness.

4.2 Variables [9]

The SIR model is examined with respect to time, hence the independent variable will be taken as time t which is measured

in days. Now moving on to the dependent variables; for these, two related variable sets will be considered.

The first variable set shows the number of people that is divided into three groups:

$S = S_t$; the number of infected individuals who are currently infected with the disease and are contagious

$I = I_t$; the number of susceptible individuals who may catch the infection if exposed

$R = R_t$; the number of recovered or removed individuals who cannot get the disease or pass it to others

The second variable set shows the fraction of population, N , that is also divided into three groups:

$s_t = S_t/N$; the susceptible fraction of the total population

$i_t = I_t/N$; the infected fraction of the total population

$r_t = R_t/N$; the recovered fraction of the total population

As we can see that both the sets are proportional to each other, therefore using either of the sets will tend to give us the same information about the pandemic. However, for convenience in calculations, the variables showing fraction of the total population, i.e. (s_t, i_t, r_t) , will be used in this investigation.

4.3 Assumptions

1) Homogeneous Mixing

The first assumption is that the model considers homogeneous mixing, which means that each and every individual who is still uninfected faces the same risk of coming in contact with an infected individual and as a result themselves being infected. This is really not the case for the COVID-19 virus, as according to WHO people who are older than a certain age, as well as people with pre-existing medical conditions such as heart disease, asthma, diabetes, etc. are generally more vulnerable to this virus [10]. However, anyhow it is assumed in this model that there will be homogeneous mixing.

2) Constant Population Size

The next assumption is that the SIR model considers population, N , to be immense and constant. This simply means that there will be whatsoever no addition or reduction in the susceptible group-set. Hence, there will no births or deaths and no immigration or emigration. As a result, according to this assumption, the only way an individual leaves S is by coming in contact with an infected individual from I and hence become a part of I itself. This again is not possible for any outbreak for that matter of fact because in any country there will always be births and deaths that don't relate to the outbreak. However, this assumption is very important for the model as a constant population is required to study the dynamics better.

3) No Latency

The third assumption is that there will be no latency, i.e. there is basically no incubation period. Basically, one immediately

gets infectious once infected. As aforementioned, there is indeed an incubation period for the SARS-CoV-2 which is for 14 days or sometimes even longer. This means that, in this case, an infected individual will start to feel the symptoms of the virus after this number of days but, for the simplification of the model, we will assume that there will be no latency.

Hence, in order to formulate the SIR model the following are elementary: 1) the individuals in S can either remain disease-free or become infected. There is no other option that the individuals in this group fall in. 2) the individuals in I will remain infectious until they are recovered completely and permanently. 3) the individuals in R once recovered or removed will never be at risk again and can be said to be immune to the disease. However, as we can see in the case of COVID-19, recovered patients have tested positive of the virus again. However, again to simplify this model, this assumption is necessary.

4.4. Mass Action Principle [11]

The mass action principle is a basic principle used in the SIR model which expresses in a mathematical formula the variables of the number of susceptible individuals and the number of infected individuals in the population. As aforementioned, the disease is spread when a susceptible comes into contact with an infective and as a result contracts the disease. The principle shows the how the number of encounters between the susceptible and the infectives is given by the product of S_t and I_t :

4.5 Changes in s, i, r

1) Changes in the susceptible class:

As aforementioned, the disease is only spread when an individual from the susceptible group comes into contact with an individual from the infected group and hence becomes infected. However, only the proportion β of the encounters between susceptible and infectives each day are regarded as infected. β is the infection rate which is a constant used in this investigation. The infection rate controls the rate of spread of the virus or the disease. The infection rate can be calculated by the formula $\beta = R_0 \times \gamma$, where R_0 is the Basic Reproduction Number, taken as 2.6 and γ which is the recovery rate is calculated by the formula $\gamma = \frac{1}{D}$, where D is the average duration of recovery D . The average duration of recovery is taken as 18 days resulting in $\gamma = 0.06$. Therefore, now β is calculated to be 0.14 [12].

Thus when considering the next time interval, the following equation can be derived:

$$s_{t+1} = s_t - \beta s_t i_t$$

2) Changes in the infected class:

After one time increment, the infected class increases by a certain amount because of the addition of the newly infected. However, there is also a reduction in the infected class which is caused when individuals in the class recover or unfortunately die and thus move on to the recovered or

removed class. Only a proportion γ of the people in the infected class really die or are recovered. This constant is the recovery rate and is already calculated to be 0.06.

Thus when considering the next time interval, the following equation can be derived:

$$i_{t+1} = i_t + \beta s_t i_t - \gamma i_t$$

3) Changes in the recovered or removed class:

As we can see that after an increase in the time interval t , γ proportion of the infected class move to the recovered or the removed class. Thus, the following equation can be derived:

$$r_{t+1} = r_t + \gamma i_t$$

4.6 Deriving Final Equations

Therefore, now we have the following equations:

$$s_{t+1} = s_t - \beta s_t i_t \tag{1}$$

$$i_{t+1} = i_t + \beta s_t i_t - \gamma i_t \tag{2}$$

$$r_{t+1} = r_t + \gamma i_t \tag{3}$$

Using these three equations a new set of equations showing changes in each of the classes can be derived as follows:

$$\Delta s = -\beta s_t i_t \tag{4}$$

$$\Delta i = \beta s_t i_t - \gamma i_t \tag{5}$$

$$\Delta r = \gamma i_t \tag{6}$$

When we add the set of equations 1-3 we get a very interesting result of:

$$s_{t+1} + i_{t+1} + r_{t+1} = s_t + i_t + r_t$$

We can also say that:

$$s_{t+1} + i_{t+1} + r_{t+1} = s_t + i_t + r_t = 1$$

This equation, for a fixed population, fits all t as any dead individuals will be counted under the recovered or removed class.

Then from adding the set of equations 4-6, we get:

$$\Delta s + \Delta i + \Delta r = 0$$

This equation also shows that net change in the total population is 0.

SIR MODEL AND THE COVID-19 OUTBREAK

Now that we have derived all the required equations to formulate and finally make the model, we will be applying it to Wuhan, China. I chose Wuhan to model as firstly, it was the origins of the virus and knowing the dynamics of the virus in Wuhan would tell us a lot about the nature of the virus in other countries too, and secondly, China is the only country which has recently been having very few daily new cases. Hence, I thought that it would be easier to compare the model to the

real life scenario easily if the disease in China is at its verge of coming to an end (this assumption is made on April 16 2020, irrespective of the further situations).

5.1 Application of the SIR Model to COVID-19 in Wuhan, China

As the second assumption for this model was that there should be a constant population size, we will take Wuhan's population as it was at the end of the year 2019. This is because first case of novel coronavirus was reported on December 31 2019. As per the report made till April 16 2020, there is no individual reported who is immune to this virus, therefore we will assume the whole population of Wuhan to be in the Susceptible class:

$$S_0 = 11,000,000$$

According to reports, on the first day, 40 cases were reported being infected by zoonotic exposure. Therefore:

$$I_0 = 40$$

And since nobody was either recovered or removed:

$$R_0 = 0$$

If we multiply these initial values with $\frac{1}{N}$, the following equations will be formulated:

$$s(0) = 1$$

$$i(0) = 3.63636 \times 10^{-6}$$

$$r(0) = 0$$

Table 2 lists all the variables used for modelling the SIR model for Wuhan:

Table 2: Variables used

Total Population	11,000,000
Δt	1
Initial Infected	40
Initial Recovered or Removed	0
β	0.14
γ	0.06

5.2 Data Collection

Now we apply the data into the SIR equations:

Day 1:

Change in the number of susceptible:

$$s_{t+1} = s_t - \beta s_t i_t$$

$$s_{t+1} = 1 - 0.14 \times \frac{11,000,000}{11,000,000} \times \frac{40}{11,000,000}$$

Therefore, $s_{t+1} = 0.999999475$

Changes in the number of infected:

$$i_{t+1} = i_t + \beta s_t i_t - \gamma i_t$$

$$i_{t+1} = 3.63636 \times 10^{-6} + 0.14 \times \frac{11,000,000}{11,000,000} \times \frac{40}{11,000,000} - 0.06 \times \frac{40}{11,000,000}$$

$$\text{Therefore, } i_{t+1} = 3.9596 \times 10^{-6}$$

Changes in the number of recovered or removed:

$$r_{t+1} = r_t + \gamma i_t$$

$$r_{t+1} = 0 + 0.06 \times \frac{40}{11,000,000}$$

$$\text{Therefore, } r_{t+1} = 2.0202 \times 10^{-7}$$

Day 2:

Changes in the number of infected:

$$s_{t+2} = s_{t+1} - \beta s_{t+1} i_{t+1}$$

$$s_{t+2} = 0.999999475 - 0.14 \times 0.999999475 \times 3.9596 \times 10^{-6}$$

$$\text{Therefore, } s_{t+2} = 0.999998903$$

Changes in the number of infected:

$$i_{t+2} = i_{t+1} + \beta s_{t+1} i_{t+1} - \gamma i_{t+1}$$

$$i_{t+2} = 3.9596 \times 10^{-6} + 0.14 \times 0.999999475 \times 3.9596 \times 10^{-6} - 0.06 \times 3.9596 \times 10^{-6}$$

$$\text{Therefore, } i_{t+2} = 4.31156 \times 10^{-6}$$

Changes in the number of infected:

$$r_{t+2} = r_{t+1} + \gamma i_{t+1}$$

$$r_{t+2} = 2.0202 \times 10^{-7} + 0.06 \times 3.9596 \times 10^{-6}$$

$$\text{Therefore, } r_{t+2} = 4.21998 \times 10^{-7}$$

5.3 Graphing the SIR Model [9]

The data calculated was put into Microsoft Excel software and was further calculated till 260 days. Consequently, a graph was plotted on the same software for $\Delta t = 1$ showing the susceptible, infected and recovered or removed fraction of the population for COVID-19 in Wuhan China. The data for the graph plotted is added in Appendix 2.

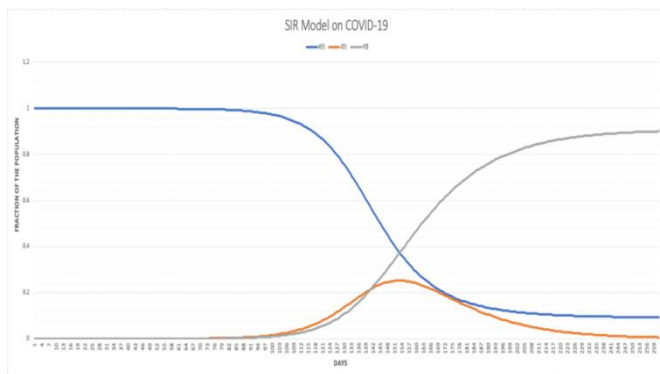


Figure 2: The SIR Model for COVID-19 in Wuhan, China

Figure 2 shows the SIR Model for COVID-19 in Wuhan, China under the aforementioned assumptions and by using the

variables table 2. We can see a drastic drop in the susceptible class or s_t after around 97 days. As shown in figure 2 we can see that s_t reaches 0.5 at around 145 days and then the curve subsequently settles at 0.1 after around 211 days. Then, we can see that the recovered or removed class, or r_t starts to increase rapidly at around day 97 and half the fraction of the population is recovered after 163 days. Gradually, the r_t curve starts to flatten at 0.9 after day 253. After studying the nature of the s_t and r_t curves, we see that the two curves tend to show an inverse relationship. Interestingly, we also see that as the recovered or removed curve reaches the highest point, the susceptible curve reaches a plateau. Then, as for i_t there are no changes until day 97, when the infected curve starts rising instantaneously and then reach a maxima after 154 days at around 0.25. Then, the infected curve gradually falls and approaches 0 at around day 250.

SOLVING THE SIR MODEL USING EULER'S METHOD

6.1 Euler's Method for solving a differential equation [13]

After applying the SIR Model to the pandemic of COVID-19, I wanted to explore a way to solve the equations to grasp a better understanding of what the variables meant in the SIR Model and also compare models of different increments and compare them with the original model as shown in figure 2. Hence, I decided on using the Euler's Method to approximate solutions to differential equations. Starting with a general first order Initial Value Problem:

$$\frac{dy}{dt} = f(t, y)$$

$$y(t_0) = y_0$$

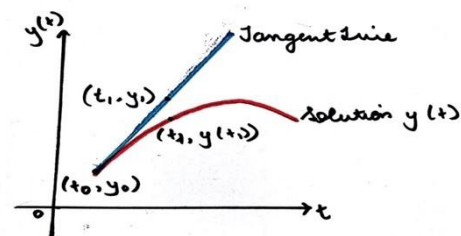


Figure 4: The SIR Model for COVID-19 in Wuhan, China for $\Delta t = 5$

To proceed further, let's approximate the solution to eq1 near $t = t_0$. We will start with two sets of information we have about the solution. Firstly, we know the value of the solution at $t = t_0$ from the initial conditions, $s(0)$, $r(0)$ and $i(0)$. Then, secondly we also know the derivative at $t = t_0$. This can be obtained by putting in the initial condition into $f(t, y)$ into the differential equation itself. There we derive:

$$\left. \frac{dy}{dt} \right|_{t=t_0} = f(t_0, y_0)$$

The equation for the tangent line in figure 3 is:

$$y = y_0 + f(t_0, y_0)(t - t_0)$$

As shown in figure 3, Euler’s Method makes linear and numeric approximations of the actual graph using small step values which are represented by t .

Using the Euler’s Method it can be derived that:

$$s_t = s_{t-1} + \Delta s$$

$$i_t = i_{t-1} + \Delta i$$

$$r_t = r_{t-1} + \Delta r$$

In order to simplify the modelling as well as the calculations, the equations representing the susceptible, infected and recovered or removed fraction of the whole population were considered. As we have already calculated the values or expressions for Δs , Δi and Δr , the following equations were deduced:

$$s_{t+1} = s_t - (\beta s_t i_t) \cdot \Delta t \tag{1}$$

$$i_{t+1} = i_t + (\beta s_t i_t - \gamma i_t) \cdot \Delta t \tag{2}$$

$$r_{t+1} = r_t + (\gamma i_t) \cdot \Delta t$$

Table 4 presents some sample calculation for the equations, with a time increment Δt , which were derived above. In this investigation, the Euler’s Method has been used to model for two values of Δt , i.e. 5 and 10. The models can be seen in figure 4 and 5.

Δt	s_t	i_t	r_t
$t=0$	$s_0 = 1$	$i_0 = 3.63636 \times 10^{-6}$	$r_t = 0$
$t=1$	$s_1 = s_0 - (0.14s_0i_0) \cdot 1$	$i_1 = i_0 + (0.14s_0i_0 - 0.006i_0) \cdot 1$	$r_1 = r_0 + (0.06i_0) \cdot 1$
...
$t=n$	$s_n = s_{n-1} - (0.14s_{n-1}i_{n-1}) \cdot \Delta n$	$i_n = i_{n-1} + (0.14s_{n-1}i_{n-1} - 0.006i_{n-1}) \cdot \Delta n$	$r_{n-1} = r_{n-1} + (0.06i_{n-1}) \cdot 1$

The equations solved with Euler’s Method are usually used and solved with specific computer software in which estimations and repetitions of the steps can be done easily and much quicker than humans.

6.2 T-step-value in Euler’s Method for solving a differential equation

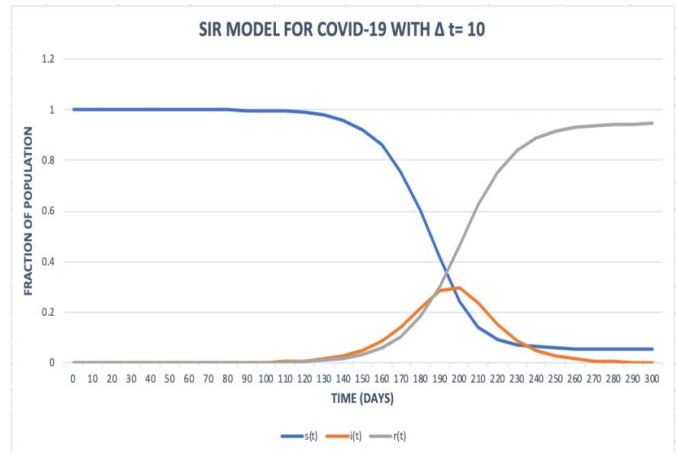


Figure 5: The SIR Model for COVID-19 in Wuhan, China for $\Delta t=10$

Figures 4 and 5 show two models with $\Delta t = 5$ and $\Delta t = 10$ respectively.

After seeing the models in figures 2, 4 and 5 we can deduce that depending on the step size of t -value, the graphs show slightly different results. Starting with the susceptible class, in figure 4 we can see that there is a drastic drop in the s_t class at day 105 and levels off at day 240 below 0.1 fraction of population. Consequently, in figure 5, there is a fall in s_t curve at day 120 and levels off at day 230 well below 0.1 fraction of population. Then, for the recovered or removed class, there is a sudden increase in the r_t curve in figure 4 at day 105 which levels at its maximum around day 255 above 0.9 fraction of population. However, in figure 5 we see that the r_t curve starts to increase at day 120 and levels off at day 260, which is well above 0.9 fraction of population. Finally, for the infected class, in figure 4 we can see that the i_t curve starts to increase after day 105, then, reaches its maximum on day 170 at around 0.3 fraction of population, and then finally decreases till day 255, after which the curve levels off at 0 fraction of population. However, as shown in figure 5, the i_t starts to increase at day 120, then reaches its maximum point on day 200 at 0.3 fraction of population, and finally decreases till day 270 and then levels off at 0 fraction of population.

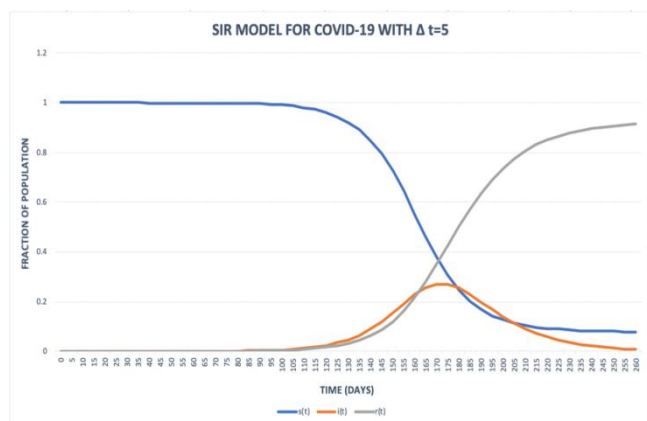
When comparing the three models of $\Delta t = 1, 5, 10$ we can see that there some slight differences in some important features of the model. Firstly, the curves in all three figures start showing changes after different number of days; for example, when the step-value is 1, the curves show a change after 97 days, when it is 5, the curves show a change after 105 days and finally, when it is 10, the curves show a change after 120 days. Then, secondly, the end points of the s_t and r_t curves also differ with the step-value. When $\Delta t = 1$ the s_t curve levels off at 0.1, however, when Δt equals 5 and 10, the s_t curve levels off just below 0.1 (around 0.075) and around 0.05, respectively. Similar is the case with the recovered or removed class, when $\Delta t = 1$ the r_t curve levels off at 0.9,

however, when Δt equals 5 and 10, the r_t curve levels off just above 0.9 (around 0.925) and around 0.95, respectively.

Hence, I deduced that in order to have an appropriate and accurate model of the actual situation, the t-step-value has to be carefully chosen keeping in mind specific parameters such as the population size and the range of time. For some fast spreading or highly contagious diseases, the t-step-value should be kept very low, it can be lesser than one day and sometimes even some hours. However, on the other hand, too small a t-step-value could make the process of finding the values for s_t , i_t and r_t too time consuming and inefficient as only very minute results will be shown (this is only possible in slow-spreading diseases).

CONCLUSION

The classical SIR epidemiological model is one of the earliest and most important results of mathematical biology. Although the SIR model is used by epidemiologists all across the world, it still carries many potential drawbacks. Some of the drawbacks have already been mentioned in section 4.3 but I will be reiterating them here and looking into them with greater depth. Firstly, the model assumes that the population will be homogeneously mixed. This basically means that each individual has an equal probability or likeliness of contracting the disease. However, this is not the case for COVID-19 as primarily, it is proven that the older population with medical diseases or illness, have the greatest probability of being infected, and secondly, each individual will not have the same number of contacts with every other individual. Then, the model also assumes a constant population size.



This is practically impossible as at any point of time, there will be births and also deaths that might not be because of the disease. People also may immigrate between different cities and provinces, if not countries. Moving on, the model also assumes no latency, which basically means that there is no incubation period, however, as aforementioned, the mean incubation period for COVID-19 two weeks. Finally, the SIR Model also assumes that once an individual is recovered, he/she is *immune* to the disease and cannot contract the disease again. As mentioned in section 4.3, numerous countries have reported recovered cases of COVID-19 testing positive again.

To fix this problem, the SIS model (Susceptible-Infected-Susceptible) could be devised. These specific assumptions bring a large amount of uncertainty to the model and makes it less reliable.

The results in sections 5.3, and 6.2 for Wuhan, China can be used as a reference when generally studying the dynamics of the model and determining the amount of days till the disease has been terminated or has been temporarily finished. A lockdown or social distancing can have enormous effects on the dynamics of the pandemic as it affects the infection rate and thus the model can be altered accordingly. This feature of the model can be used by the government to plan effectively on the duration of these measures as for the betterment of the population. To gain a better understanding, a renewed and better form of the SIR model, known as the SEIR (Susceptible-Exposed-Infected-Recovered/Removed) model could also be used. Other forms of the SIR model like the MSIR, SIRS or SQIR model could also be used.

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